De Fermat and the method of '*Descente infinie*'
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Abstract

Another proof of de Fermat's last theorem is presented..
A proof of de Fermat's last theorem is obtained by using '*Descente infinie*'.

§1. Introduction.

'Fermat's equation: $x^n + y^n = z^n$, **(1.1)**

has no solutions for $n \geq 3'$.
A statement by Andrew Wiles written on the black board after the presentation of the conclusions concerning equation **(1.1).**
Is there another proof of the last theorem of de Fermat, fitting into the margin of de Fermat's copy of Diophantus?
de Fermat mentioned his proof not to fit into the width of the margin of his Diophantus copy.
An Ockham's razor, sort of. This stimulated me to investigate de Fermat's last theorem.
It all started with $n = 2$ and

$$a^2 + b^2 = c^2. \quad \textbf{(1.2)}$$

For equation **(1.2)** there are an infinite number of solutions for a, b and c as positive integers; $\{a, b \ and \ c \in \mathbb{P}\}$. The Pythagorean Triples.
This equation, **(1.2)**, is discussed in some detail in Noordzij.

For $n = 3$ and $n = 4$, the proof of the non-existence of integer solutions has been given with the so-called method of '*Descente infinie*' (Giorello, G., et al).

§2. Notation

I assume a solution of **(1.1)** in terms of integers x, y and z : a so-called Fermat Triple.
I denominate the solution a "Fermat" triple: $x = A, y = B$ and $z = C, [A, B, C]$

A, B and C being positive integers. A is an odd and B is assumed to be even and C must be odd.
So, $\{\exists A, C \in \mathbb{P}|$ and odd$\}$ and $\{\exists B \in \mathbb{P}|$ and even$\}$; $\{\forall n \in \mathbb{P} | n \geq 3\}$.
A, B and C are relative- or co-prime.
I will use(see e.g., Spivak):
$\forall$, the al kwantor meaning : "for all". So, $\{\forall n \in \mathbb{P}\}$ →for all n belonging to the set of natural numbers larger than zero.
$\exists$, the existential kwantor: "there exists". So, $\{\exists n \in \mathbb{P}\}$ →there exists a n belonging to the set of natural numbers larger than zero.

§3. de Fermat's last Theorem.

§3.1 $n = 3$, $n = 4$

With the method of "*Descent Infinie*", it has been proven no Fermat Triples to exist for $n = 3$, and $n = 4$. I shall use these results.
First $n = 3$.
There exists no Fermat triple for $n = 3$. I shall make use of this result to proof no Fermat triples to exist for integer multiples of $n = 3 \rightarrow n = m \cdot 3$.
So,
$\{\forall\ m \in \mathbb{P} | n = m \cdot 3\ \}$ there exists no Fermat Triple.

Proof 1

$\{\exists [A, B, C] \in \mathbb{P}\}$.
We have, with $n = m \cdot 3$,
$A^n + B^n = C^n \Leftrightarrow A^{m3} + B^{m3} = C^{m3} \Leftrightarrow (A^m)^3 + (B^m)^3 = (C^m)^3$,
where $\{A \in \mathbb{P}\}$.
With the result of the 'Descente infinie' (Giorello, G., et al) for $n = 3 \rightarrow$
$\rightarrow \{\nexists (B^m, C^m) \in \mathbb{P}\}$.
So, a for a given integer $A \rightarrow \{B^m, C^m \notin \mathbb{P}\}$.
Consequently, $\{B, C \notin \mathbb{P}\}$.
This contradicts $\{\exists (B, C) \in \mathbb{P}\}$.
Now, suppose for a given $\{A \in \mathbb{P}\}$ and $\{\exists (B, C) \in \mathbb{P}\} \Longrightarrow \{B^m, C^m \in \mathbb{P}\}$.
This contradicts the result of the 'Descente infinie'.
Hence, $\{\nexists [A, B, C] \in \mathbb{P} |\ n = 3 \rightarrow n = m \cdot 3\}$.
End of proof.

Next, we use $n = 4$ and the non-existence of Fermat Triples resulting from the "*Descent Infinie*".
As mentioned in the Introduction, there exists no Fermat triple for $n = 4$. I shall make use of

this result to proof no Fermat triples exist for integer multiples of $n = 4 \rightarrow n = m \cdot 4$.

$A^n + B^n = C^n \Leftrightarrow A^{m4} + B^{m4} = C^{m4} \Leftrightarrow (A^m)^4 + (B^m)^4 = (C^m)^4$.

For a given integer $A \rightarrow \{(B^m, C^m) \notin \mathbb{P}\}$.

Completely like the above proof for $n = 3$:

$\{(B, C) \notin \mathbb{P}\} \rightarrow \{\nexists[A, B, C] \in \mathbb{P}|\ n = 4 \rightarrow n = m \cdot 4\}$.

Hence for the set of integer multiples of $n = \{3,4\}$, there are no Fermat triples.

Now, let's look again at

$A^{m3} + B^{m3} = C^{m3}$,

and $\{\forall m \in \mathbb{P}\}$.

For a given integer $A \rightarrow \{\nexists[A, B, C] \in \mathbb{P}|n = 3 \rightarrow n = m \cdot 3\}$.

Proof 2

$A^n + B^n = C^n \Leftrightarrow A^{m3} + B^{m3} = C^{m3}$.

For a given integer $A \rightarrow \{B, C \notin \mathbb{P}\}$. See the above proof 1 for $n = m \cdot 3$.

Now,

$A^n + B^n = C^n \Leftrightarrow A^{m3} + B^{m3} = C^{m3} \Leftrightarrow (A^m)^3 + (B^m)^3 = (C^m)^3 \Leftrightarrow (A^3)^m + (B^3)^m = (C^3)^m$.

Again, Proof 1: $\{\forall A \in \mathbb{P}\} \rightarrow \{\nexists B, C \in \mathbb{P}\}$.

Furthermore, in Proof 1 we obtained for a given integer $A \rightarrow \{B^m, C^m \notin \mathbb{P}\}$.

Then,

$\{B^3, C^3 \notin \mathbb{P}\} \rightarrow \{\forall m \in \mathbb{P}\} \rightarrow \{\nexists[A, B, C] \in \mathbb{P}\}$ for $\{\forall n \in \mathbb{P}|n \geq 3, n = m \cdot 3\}$.

End of Proof

No new information is created? Let us have a look.

In the above Proof 2, we made use of a Proof 1.

Then, we have

$(A^3)^m + (B^3)^m = (C^3)^m$.

We have proven for a given integer $A \Longrightarrow \{B^3, C^3 \notin \mathbb{P}\}$.

Now, set $m = 2$,

$[A^3, B^3, C^3]$,

can constitute a set of Pythagorean Triples.

Hence, B and C can be integers.

However, we have proven above, Proof 1 and 2

$\{\nexists[A, B, C] \in \mathbb{P}|\ n = 3 \rightarrow n = m \cdot 3\}$.

So, the Pythagorean Triples $[A^3, B^3, C^3]$ does not represent a set of integers:

$\{\nexists[A, B, C] \in \mathbb{P}\}$.

Then, B and C cannot be integers and $[A^3, B^3, C^3]$, cannot constitute a set of Pythagorean Triples.

Next for $n = 4$ completely simarlily,

$[A^2, B^2, C^2]$ does not represent a set of integers: $\{\nexists[A, B, C] \in \mathbb{P}\}$.

Then, B and C cannot be integers and $[A^2, B^2, C^2]$, cannot constitute a set of Pythagorean Triples.

Furthermore,

$A^3 \equiv D, B^3 \equiv E$ and $C^3 \equiv F$,

$(D)^m + (E)^m = (F)^m$.

Then,
$\{E, F \notin \mathbb{P}\}$, for $\{\forall m \in \mathbb{P} | m \geq 3\}$.
So, no Fermat Triples exist.
In the following section I will use the results of $n = 3$.

§3.2 $n > 3$.

$a^n + b^n = c^n$.

The preceding equation can be written as:
$(a^{n/3})^3 + (b^{n/3})^3 = (c^{n/3})^3$.
With the result of the '*Descente infinie*' (Giorello, G., et al) for $n = 3 \rightarrow$
$\rightarrow \{\nexists (b^{n/3}, c^{n/3}) \in \mathbb{P}\}$.
Consequently,
$\{\nexists (b, c) \in \mathbb{P}\}$

Proof
$\{\nexists (b, c) \in \mathbb{P}\}$.
Suppose
$\{\exists (b, c) \in \mathbb{P}\}$.
Then,
$\{\exists (b^n, c^n) \in \mathbb{P}\}$.
So, the possibility exists
$\{\exists (b^{n/3}, c^{n/3}) \in \mathbb{P}\}$.
This possibility contradicts the method of the '*Descente infinie*'.
End of Proof.

§4. Conclusion.

For $n = m \cdot 3$ and $n = m \cdot 4$, there do not exist Fermat Triples.
This analysis is used to finally obtain the nonexistence of a Fermat Triple, $\boldsymbol{n \geq 3}$, has been proven in §3 using the method of the '*Descente infinie*' (Giorello, G., et al).
For further reading on de Fermat's last theorem I like to mention Simon Singh's book.
Finally, to conclude the above approach on de Fermat's last theorem I also cite Feynman on de Fermat's last theorem: "*For my money Fermat's theorem is true*". Feynman estimated that the probability of finding integer solutions is less than 10^{-200} (Schweber).

§5. Literature:

Giorello, G. and C. Sinigaglia, *Fermat- I sogni di un magistrato all'origine della matematica moderna*, 2001, Le Science, Milano.

Schweber, Silvan S., *QED and the Men Who Made it: Dyson, Feynman, Schwinger and Tomonaga,* 1994, Princeton University Press.

Singh, S., *Fermat's last theorem*, 2005, Harper Perennial.

Spivak, D.I., *Category Theory for the Sciences*, The MIT Press, 2014.