Incompleteness, Uncertainty and Archimedes 'Lever'.

Update 2022-12-08, §15 Artificial Intelligence, The Juvenalis Dilemma.

The Ultimate Lever: Know Thyselve

Use Ockham's razor to make the lever work

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Content

Tags: Archimedes Lever, Education, Gödel, History, von Mises, Roth, American Constitution, Multiverse, Infinity, Goldbach, Fermat, Artificial Intelligence, Factorials.

1 Gödel and von Mises.

In 1931 the Austrian, later on the American, mathematician Kurt Gödel published his Incompleteness Theorems (1). The Building of the German mathematician Hilbert collapsed with the Incompleteness Theorems .

In 1922 the Austrian Ludwig von Mises, later on American(?), published 'Die Gemeinwirtschaft'. This book has been translated into the English language in 1951, the title: Socialism. An economic and sociological analysis'', New Haven Yale University Press.
A striking quotation from this book: "*To demand that Law should have arisen legally is to*

demand the impossible. Whoever does so applies to something standing outside the legal order a concept valid only within the order".

Did Gödel and von Mises met and discus incompleteness? May be. Morgenstern was a close friend of Gödel. Morgenstern, Von Neuman and Morgenstern, studied economics with Ludwig von Mises. Von Mises studied with Carl Menger.

2 Gödel and Philip Roth and the natives.

In the series *"Hidden in Plain Sight"* Thomas presents the reader with the story of Einstein promoting Gödel to become American. Gödel had found a loophole in the American constitution resulting in the possibility of dictatorship in America. Isaacson described this in more detail on page 510 of his biography on Einstein. The Framers thought they constructed a system to prevent dictatorship. Could they have listened to Gödel. Einstein prevented Gödel to present his finding(Isaacson).
Was Philip Roth aware of the incompleteness of the constitution when he wrote *The Plot Against America* ? Well, something came through in 2016.
Philip Roth (page 307): "Rising steadily into the stream of a warm, gentle tailwind, the most famous small plane in aviation history—the modern-day counterpart of Columbus *Santa Maria* and the Pilgrims' *Mayflower*—disappears eastward, never to be seen again."
L'histoire se répète, not exactly in the same way?
On page 309 Roth wrote about a radio announce by the Third Reich of the disappearance of Lindbergh. The way the Nazis dealt with the disappearance sounds rather familiar in 2017.
America First and the natives. Well, in this respect the movie picture *"Gangs Of New York"* is quite an education indeed. It is a sort of law of nature the sophomore immigrants most violently oppose the freshmen immigrants. Isn't it Herr Trump(Putin's plaything) or Signor Trump, dependent on the chin configuration?
What about room 101? Well, the Russians did not need 101. Trump was and has been hollow and could easily be filled with Russian material. Alexander Hamilton(1788) knew Trump was coming (Snyder). In October 2018 the "native" Trump announced planning an executive order. This order apparently instruct federal agencies to refuse to recognise the citizenship of children born in the United states if their parents are not citizens (The Atlantic, October 2018). Well, Herr Trump why not elongated your lever and include grandparents?

3 The Theory of Everything.

Similar to Gödel's proof of the non-existence of a closed system of axioms (evidence of absence), there is not such a thing like the "Theory of Everything" . You always need something like an Archimedean Lever. In the title of the book by Kraus : *"A Universe from Nothing: Why There Is Something Rather than Nothing*". Again, a lever?
In the book series *"Hidden in Plain Sight"* Thomas wrote: "A theory that cannot be refuted…….". Well, such a theory cannot be falsified. How can it possible be to find our laws of nature based on a theory that cannot be falsified? "There is nothing outside the universe"

is such a principle. It cannot be falsified. It all depends on the definition of “Universe”. Again, a job for Archimedes ‘Lever.
Thomas is quite right in rejecting the multiverse approach. Anything goes, so nothing is really proven by using multiverses to save string theory. Thomas used just a few formulas. Then it is a pity to see a formula for the escape velocity in part 2 of The Hidden in Plain Sight series (page 78):

$\frac{1}{2}mv^2 = \frac{GMm}{r_{esc.}}$.

m is the mass of the object trying to escape and r_{esc} denoted to be the escape velocity. r_{esc} appears to be the black hole radius. A bit sloppy.

Another example of Incompleteness has been given by D’Alembert(Crépel, et al). It is about the difference between words which can be defined, and which cannot. D’Alembert explained that you need words which cannot be defined.
A classic example of incompleteness can be found in the book of Weinberg: “*To Explain the World: The Discovery of Modern Science”*. Weinberg quoted some lines of Xenophanes: “And as for certain truth, no man has seen it, nor will there ever be a man who knows about the gods and about the things I mention. For if he succeeds to the full in saying what is completely true, he himself is nevertheless unaware of it, and opinion is fixed by fate of all things”.

Gödel did not comment on the question about the relation between Incompleteness and Heisenberg’s Uncertainty relations

Where does the mass of the Higgs ' boson come from? What generates mass? This seems to be a kind of juvenal Dilemma, the incompleteness theorem of Gödel and Archimedes’ lever. Thomas also paid attention to the subject of the mass of Higgs’s boson:
“Hidden in Plain Sight 7”.

Remark: A few remark on number *7*: pages 26 on Bayes theorem: Thomas writes " So mathematical probability gives us a way of moving from an effect(the throwing of the dice) to predicting a likely cause (the result of the throw). This sentence I do not understand. Probability moves from cause to effect I assume?
Page 82: " All the elements heavier than the chemically inert gases of hydrogen......"? Hydrogen chemically inert?

The observation influences the result. If the experiment is redone, we find a slightly different observation. This has become especially apparent in Quantum mechanics. Measurements affect observation is also reflected in the so-called α and γ sciences. Although experiments are performed on a large object (person or group of people), the object changes due to the measurement. With the fever thermometer the body temperature will not be affected appreciably. However, a measurement on the neuron network in our brains will be influenced by that measurement. Can the principles of Quantum mechanics be applied? How to apply this on the neuron network?

A nice experiment is described in the magazine Intelligent Life (now the Economist 1843) of May/June 2014: *Thinking*. In this story the effect of exposure is described with the example of the Mona Lisa: "Once a thing becomes popular, it will tend to become more popular still". But Intelligent Life does not leave it here and mentions: ".. Exposure effect does not work the same way on everything ". Repeat the experiment often and "... About time, exposure favors' the greater artist". "Repeat the experiment often" seems at least the experimental reality in Quantum mechanics and Particle Physics.
Another study, described in the Economist from May 3, 2014, on Pain Perception, describes how the environment of this research affects the results of the experiments. When this happens in physics, the research is repeated, and the environmental influence is excluded.
Problem: How to perform experiments with humans and animals meaningfully?
Still, Particle Physics is also in need of a lever (Smolin).

4 Back to the future with the multiverse.

According to Weinberg(Chapter 11) science really took off in the sixteenth century. Before that science was mixed with religion. Religion what now we call philosophy.
On page 163 of chapter 11, "*The Solar System Solved"*, Weinberg refers to today's theoretical physics. Here he mentioned the subject of multiverse, sub universes and the anthropic principle. (See for the subject of multiverse e.g., Susskind).
With introducing the idea of multiverse in the section of his book where Weinberg paid attention to Galileo, Weinberg moves back to the philosophes. Philosophes despised by Galileo, to say the least. Why did Weinberg do that? I don't know. With adopting the multiverse anything goes. As a consequence, nothing goes.
The discussion on subject matter appears to me almost a religious discussion(Ananthaswamy).
If we need the multiverse to understand our universe, the anthropic principle, we could also denote the multiverse to be what we now know to be our universe. Furthermore, the universe as part of the multiverse is considered to be our planet.
I can live and feel quite well with: *We living things are an accident of Space and Time*. This is the title of the paper by A. Lightman in the Atlantic, Science section, September 2022.
Conjecture:
The anthropic principle and the multiverse contradicts evolution to say the least.

A minor remark on the book of Weinberg: On page 205 he mentioned the mathematician, physicist, and astronomer Willebrord Snell or Snellius , the Dane. Well, observed from the US, Denmark and The Netherlands are one and the same. Europe, you know. For the sake of completeness, I like to mention in the days of Snellius The Netherlands did not exist. There were Flanders and The Seven Provinces. Snellius worked at the university of Leiden.
In the same time frame Simon Stevin was a taxman in Bruges and after the troubles started with the Spaniards(the eighty years' war), Stevin moved to the University of Leiden in the Province of Holland. This province was part of The Seven Provinces. Stevin became an adviser to the stadhouder-prince Maurits(Pye, pages 309-310).

5 Two-dimensionality, simulation, and the holy grail.

It started in the first half of the 20th century with the $E = mc^2$ and all is relative Sehnsucht. Followed by *The dancing Wu-Li Masters* and *The Tao of Physics*.
Now we are on the way of further improving. It is all mathematics.
Let us start with mathematics. In 1960 Wigner concluded his paper on *The Unreasonable Effectiveness of Mathematics in the Natural Sciences* : "…….*Let me end on a more cheerful note. The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. We should be grateful for it and hope that it will remain valid in future research and that it will extend, for better or for worse, to our pleasure, even though perhaps also to our bafflement, to wide branches of learning"*. Wigner , the brother-in-law of Dirac.
Dirac on the prediction of the positron: *"My equation was smarter than I was"*.
70 years later on.
More recently Lederman, et al.,: "Nature seems to speak the language of mathematics."
All Platonian indeed. Is knowledge of Mathematics as we know it now of evolutionary advantage(Artstein)?

With this harmony of physics and mathematics come the Anthropic Principle, the Holographic Principle, and the universe as a quantum computer(Smolin).
Hence the conclusion: our world is really two dimensional and we experience three dimensions by holographic simulation. For further reading I refer for these Principles to Susskind and will concentrate on the mathematics. Since two dimensionality creates wonderful mathematics: Complex Functions(Chisholm and Morris). There you will find all we need: how to deal with poles(singularities). These singularities and branching creates the opportunity to be transferred into another database. All is information you know. So, starting on one branch and via a singularity "disappear" on another. Well, take care. You should be aware of the contour of the singularity going to zero. So, the job has to be done the singularity has still a dimension of the $(PlanckLength)^2$. This kind of simulation(illusion?) can be repeated many times. Hence, no longer a fear of dying. Dying is just a simulation or illusion. Finally, the holy grail.
For us, flatlanders, the holographic simulation fits perfectly. That is where mathematics are for.
There are a few questions left, such as how is the simulator simulated? We need a sort of Turing test for this. Is the illusion simulated or is the simulation an illusion? After about 3 billion years of evolution are there no bugs, a few bugs, or a lot of bugs in the simulation? How can the simulated flatlander living in a hologram find out? A lot of data has to be mined. We will find out.

6 From evolution to illusion.

In section 5 we learned mathematics to be the key for live. We are just two-dimensional, and the holographic principle leads the way. It is all about illusion and information. So, we have to learn to live with evolution to be an illusion. Why not?

7 Lost in Singularities.

In section 5 flatlanders are not so much lost as saved by singularities. Disappearing in nothingness and reappearing in another flatland. This ”nothingness” is in a sublime way illustrated by Updike in *Roger's Version*. Read the dialogue between Kriegman and Dale on what is going on with nothingness, the subject matter.

More on the physics and mathematics can be found in *A Universe from Nothing: Why There is Something Rather than Nothing* by Kraus,

8 Fermat's Lever and Goldbach's Conjecture.

In Noordzij(2013) Fermat's last theorem is dealt with.

To this end an expression has been derived which after a closer look seems to resemble Goldbach's Conjecture.

The expression reads: $P_i + P_j = 2^l P_k$.

In this expression $\{P_i, P_j \in \mathbb{P}\}$ where these prime numbers are not necessarily different. Furthermore $P_k \in \mathbb{N}$ .For example P_i, P_j and $P_k = 3$, and with $l = 1$, we have $3 + 3 = 6$. In addition, P_k can be a product of prime numbers. For example, $P_i = 13$ and $P_j = 47$, with $l = 2$ we have $P_j = 15$. The product of two prime numbers 3 and 5.

$P_i + P_j = 2^l P_k$ shows an even number can be written as a sum of two prime numbers, not necessarily different.

Actually $P_i + P_j = 2^l P_k$ can be written as $P_i + P_j = 2n$, $\{n \in \mathbb{N} | n \geq 2\}$ and $P_i, P_j \in \mathbb{P}$. Furthermore, we have $2 \leq P_i \leq n$ and $n \leq P_j \leq 2n - 2$. In addition, $P_j = 2n - P_i$. Hence we have $2 \leq P_i \leq n$ and $0 \leq n - P_i \leq n - 2$. This indicates you can always find the prime number P_i and consequently the prime number P_j for any n with $\{n \in \mathbb{N} | n \geq 2\}$.

Proof by contradiction: Suppose you cannot find a prime number P_i for $2 \leq P_i \leq n$ and $\{n \in \mathbb{N} | n \geq 2\}$. This contradicts the existence of prime numbers. Suppose you cannot find a prime number P_j for $n \leq P_j \leq 2n - 2$. This contradicts again the existence of prime numbers.

An almost trivial solution for Goldbach's conjecture is n to be a prime number. Then a solution $P_i = P_j = n$ can be found. A complete picture for $2n = 34$ is given in the table below:

$2n = 34$	$n = 17$
P_i	P_j
3	31
5	29
11	23
17	17

9 Infinity.

In “The Man Who Knew Infinity” Kanigel mentioned Ramanujan to have proven :

$\sum_{n=1}^{\infty} n = -\frac{1}{12}$.

In “Hidden in Plain Sight 6: Why Three Dimensions”, Thomas presented the proof of $\sum_{n=1}^{\infty} n = -\frac{1}{12}$. There he also mentioned Euler to have proven this in 1749. I could not find this in “*Leonard Euler, mathematical genius in the enlightment*”, Calinger. In this biography the subject matter is of course infinite series.

The basic assumption for the proof of $\sum_{n=1}^{\infty} n = -\frac{1}{12}$, is $\sum_{k=0}^{\infty}(-1)^k = \frac{1}{2}$. Well, this assumption is based on the knowledge the addition of $1 - 1 + 1 - 1 + 1 - 1$ … to be 1 or 0. So it appears to make sense to choose for the sum to be ½.

However, I think what is known the expectation value E of $1 - 1 + 1 - 1 + 1 - 1$ … to be ½. Why is that so? Tossing a perfect coin will give Head or Tail with equal probability of ½. The expectation value is based on one toss, $n = 1$,: Head, the sum is 0, Tail, the sum is 1.

$E = np$, or $E = \binom{n}{0}(\frac{1}{2})^1 0 + \binom{n}{0}(\frac{1}{2})^1 1 = \frac{1}{2}$.

Hence, the expectation value of $\sum_{k=0}^{\infty}(-1)^k = \frac{1}{2}$. No more , no less.

Thomas showed the use of $\sum_{n=1}^{\infty} n = -\frac{1}{12}$, to result in 26 dimensions in spacetime. Well, I suppose the 26 dimensions to be an expectation value. No more , no less.

10 Hardy and Ramanujan

Kanigel mentioned in “The Man Who Knew Infinity” the rational series:

$\frac{1}{1}; \frac{2}{1}, \frac{1}{2}; \frac{3}{1}, \frac{2}{2}, \frac{1}{3}; \frac{4}{1}, \frac{3}{2}, \frac{2}{3}, \frac{1}{4};$ …. (I put the semi colon between the subsets).

In his book on pure mathematics Hardy introduced the above series on the first page of Chapter I, first paragraph, example 4.

“The positive rational numbers may be arranged in the form of a simple series as follows:

$$\frac{1}{1}; \frac{2}{1}, \frac{1}{2}; \frac{3}{1}, \frac{2}{2}, \frac{1}{3}; \frac{4}{1}, \frac{3}{2}, \frac{2}{3}, \frac{1}{4}; \ldots .$$

Show that p/q is the $[\frac{1}{2}(p+q-1)(p+q-2)+q]$th term of the series.

[In this series every rational number is repeated indefinitely. Thus 1 occurs as $\frac{1}{1}, \frac{2}{2}, \frac{3}{3}$,…..We can of course avoid this by omitting every number which has already occurred in a simpler form, but then the problem of determining the precise position of p/q becomes more complicated.]”.

I assume Hardy meant with “show” just that and not “prove”.

$p, q \in \mathbb{N}$ and $\frac{p}{q}$ rational.

Well, set $\frac{p}{q} = \frac{3}{1}$. Substitute this value of $\frac{p}{q}$ into $\frac{1}{2}(p+q-1)(p+q-2)+q$ and you will find the fourth position.

Let us try to prove *Show that p/q is the $[\frac{1}{2}(p+q-1)(p+q-2)+q]$th term of the series.*

Proof:

$(p+q-1)$ is even then $(p+q-2)$ is odd and the other way around.

With the semi colons plugged into the above series of rational numbers you will notice $p+q$ is increasing subset wise. The set starts with $p+q=1$, a subset with one element. For the next subset we have $p+q=3$, a subset with two elements, and so on.
The element in the subset with the largest denominator is the largest term in the subset.
$[\frac{1}{2}(p+q-1)(p+q-2)+q]$ can be written as
$\frac{1}{2}[(p+q)^2-3(p+q)+2]+q.$ (7.1)
The expression between brackets in (7.1) is 0 for $p+q=2.\frac{p}{q}$ the lowest rational number in the set.
Let us define a set S and a subset $S_k=\frac{p}{q}$, where $k=p+q$ or $q=k-p$ and $k\geq 2, k\in\mathbb{N}$
A better formulation is to denote S_k to be a partition of S. S_k and S_j are disjoint for $j\neq k$. So $S_k\cap S_j=\emptyset$ and S_k is called a cell.
(7.1) shows for a given subset $k=p+q$ is a constant. Hence the value of q determines the ranking in the subset. The largest value of q and consequently the smallest possible value of $p(=1)$ gives the highest ranking, i.e., $k-1=q$ gives the highest ranking
(7.1) can also be written, with $k=p+q$, as : $\frac{1}{2}[k^2-3k+2+2q]$. (7.2)
Now we need to prove the highest-ranking rational number in S_{k-1} to be smaller than the lowest ranking number S_k. We also need to prove the lowest ranking in S_{k+1} is higher than the highest ranking in S_k.
As proven above, in S_k, $q=k-1$ gives the highest ranking. From (7.1) it also follows $q=1$ gives the lowest ranking. So, in S_k there is $1<l<k-1$, $l\in\mathbb{N}$. Substitute this inequality into (7.2), k^2-3k+2 to be a constant,
$[k^2-3k+2+2q]<[k^2-3k+2+2l]<[k^2-3k+2+2(k-1)]$. After some rearranging we have $1<l<k-1$. This necessary but not sufficient. To complete the proof of ranking in the cell S_k we have to prove for the ranking :
$1<l-1<l<l+1<k-1$. Substitute $l-1, l$ and $l+1$ respectively in (2) for q and you will find $2(l-1)<2l<2(l+1)$ where the constant k^2-3k+2 has been omitted.
We prove the highest ranking rational in S_{k-1} to be smaller than the lowest ranking rational in S_k:
The highest ranking rational in S_{k-1} is found by substitution of $q=k-2$ into (7.2). The lowest ranking rational in S_k is found by substitution $q=1$ into (2) and we have to prove $[(k-1)^2-3(k-1)+2+2(k-1)]<[k^2-3k+2+2]$. After rearranging we have $0<4$.
Now we have to prove the highest ranking rational in S_k to be smaller than the lowest ranking rational in S_{k+1}:
The highest ranking rational in S_k is found by substitution of $q=k-1$ into (7.2). The lowest ranking rational in S_{k+1} is found by substitution $q=2$ into (7.2) and we have to prove $[k^2-3k+2+2(k-1)]<[k^2-3k+2+4]$. After rearranging we have $0<4$.
Remark: As mentioned above, Hardy stated: "........*Thus 1 occurs as* $\frac{1}{1},\frac{2}{2},\frac{3}{3}$*,.....We can of course avoid this by omitting every number which has already occurred in a simpler form, but*

then the problem of determining the precise position of p/q becomes more complicated".
In looking at the sets and subsets you will notice $\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \dots$ Occur for k to be even. So, we could consider two cases: k is even and k is odd. For k is odd the analysis as given above can be followed. For k is even we omit in the subset $k = p + q = 2q$. Now you have to do the analysis given above twice. First for the left-hand side of $\frac{p}{q} = 1$ in the subset and the second time to the right of $\frac{p}{q} = 1$. A bit more complicated indeed.

11 Factorials, Binomial Distribution, and the chance of Games

The binomial theorem is:

$$(a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k. \quad (8.1)$$

This theorem can be found in many textbooks. What happens with $a = b = 1$? Well, plug this into (8.1) and you will find:

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n. \quad (8.3)$$

Do we have any practical use for an expression like Eq. (8.3)? Well, maybe when writing a code with factorials included. In that case you can test your code with help of

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n.$$

Let's have a closer look at the Bernoulli trials-the binomial distribution.
We denote p to be the probability of success and $q = 1 - p$ the probability of failure. For a binomial experiment the probability of k success is given by

$$P(k) = \binom{n}{k} p^k (1 - p)^{n-k}, \quad (8.4)$$

and consequently $\sum_{k=0}^{n} P(k) = (p + q)^n = 1$.
For example tossing a fair coin, p is the chance of Heads, say, $p = \frac{1}{2}$.
Plug this into eq. (8.4) and $P(k) = \binom{n}{k} 2^{-n}$. With $\sum_{k=0}^{n} P(k) = 1$and we have again eq. (8.3).
The binomial theorem is proved in textbooks. However, just for the fun, let's find out $\sum_{k=0}^{n+1} \binom{n+1}{k} = 2^{n+1}$. The left-hand side of this expression can be written as :

$$\sum_{k=0}^{n+1} \binom{n+1}{k} = \sum_{k=0}^{n} \binom{n+1}{k} + 1. \quad (8.5)$$

Another theorem for factorials is: $\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$. Substitute this into (8.5).

Hence $\sum_{k=0}^{n} \binom{n+1}{k} + 1 = \sum_{k=0}^{n} \binom{n}{k-1} + \sum_{k=0}^{n} \binom{n}{k} + 1. \quad (8.6)$

The second expression on the right-hand side equals 2^n, eq. (8.3).
Now we have to work on $\sum_{k=0}^{n} \binom{n}{k-1}$.

$\binom{n}{k-1} = \frac{n!}{(k-1)!(n-(k-1))!} = \frac{kn!}{k!(n-(k-1))!}$. Since $k = 0$ does not contribute

$$\sum_{k=0}^{n} \binom{n}{k-1} = \sum_{k=1}^{n} \frac{kn!}{k!(n-(k-1))!}.$$

We substitute into this expression $k - 1 = s$.
Then we have $\sum_{s=0}^{n-1} \frac{(s+1)n!}{(s+1)!(n-s)!} = \sum_{s=0}^{n-1} \frac{n!}{s!(n-s)!}$.

The right-hand side of this expression is plugged into the right-hand side of (8.6):

$$\sum_{s=0}^{n-1} \frac{n!}{s!(n-s)!} + \sum_{k=0}^{n} \binom{n}{k} + 1. \tag{8.7}$$

We know 1 to be $\frac{n}{n}$,

so $\sum_{s=0}^{n-1} \frac{n!}{s!(n-s)!} + \frac{n}{n} + 2^n = \sum_{s=0}^{n} \binom{n}{s} + 2^n = 2^n + 2^n = 2^{n+1}$.

Et Voilà. That's what we were looking for.

12 Another Archimedean Lever.

In "Hidden in Plain Sight 9 " Thomas cited Einstein. I will repeat it here:

"No problem can be solved from the same level of consciousness that created it."

– Albert Einstein.

13 A Factorial Paradox and Infinity

Normalizing the radial part of the wave function of the hydrogen atom, I dealt with the integral:

$\int_0^\infty x^n e^{-x} dx$,

where

$\{n \in \mathbb{N} | n \geq 0\}$.

With integration by parts and induction, the integral is:

$\int_0^\infty x^n e^{-x} dx = n!$.

This is called the factorial function and $n!$ is infinite for

$n = -1, -2, -3, \ldots$.(Chisholm and Morris).

So, e.g., $(-1)!$ is not defined.

Well, let's look into the series expansion of e^x for all finite values of x .

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{1}{2}x^2 + \cdots, \tag{14.1}$$

and

$$\frac{de^x}{dx} \ (= e^x) = \sum_{n=0}^{\infty} \frac{x^{n-1}}{(n-1)!} = \sum_{n=1}^{\infty} \frac{x^{n-1}}{(n-1)!} = \sum_{m=0}^{\infty} \frac{x^m}{m!}\ . \tag{14.2}$$

So,

in (14.2) I deleted the "0" term and started the series with $n = 1$.

Basically, I deleted:

$$n = 0 \rightarrow \frac{x^{-1}}{(-1)!} = 0\ . \tag{14.3}$$

Hence, for all finite values of x, (14.3) results into

$$1 = 0 \cdot (-1)!\ . \tag{14.4}$$

There is more. For example, let's analyse the following expression:

$n\binom{n-1}{n}$,

and set $n = 0$.
So,
$$0\binom{0-1}{0} = 0\binom{-1}{0} = 0 \text{ or is not defined?}$$

On the other hand:
$$n\binom{n-1}{n} = \binom{n}{n} = 1 \text{ for } n = 0, 1, 2, \dots \ .$$
Now, here is a paradox:
$$0\binom{-1}{0} = 1\ ,\ 0\binom{-1}{0} = 0 \text{ or } \ 0\binom{-1}{0} = \text{undefined.}$$
Strogatz, page 13, "There is no consistent way to define infinite times zero, …….".
There is more. In section 9 I presented the Ramanujan series: $\sum_{n=1}^{\infty} n = -\frac{1}{12}$,
where $\{n \in \mathbb{N}\}$.
Is this about anything goes? Above the integral $\int_0^\infty \frac{1}{x} e^{-x} dx$ seems to be undefined.
Strogatz, page 14, "The faint at heart say the answer is undefined, but the truth is it's infinite".
Is there a possibility to restrict 'infinite' to a certain range of infinities?

14 The Fall of the Roman Empire

A longitudinal case of innovation.
James Fallows(The Atlantic) described the fall of the fall of the Roman Empire in terms of creative destruction(Schumpeter). In this case, the Barbarians at the gate operated like an Archimedes lever to stimulate the process of innovation.
The title of Fallows article is a concise summary: *The End of the Roman Empire wasn't that bad (May be the American one won't be either).*
Much more can be read about the subject matter in *A Study of History* by Toynbee.

15 Artificial Intelligence, an Oxymoron?

Artificial Intelligence(AI) is based on available data. Consequently, AI is based on the past. Nothing wrong with that. Using AI, data can be analysed and applying a lot of criteria new data can be created. However, keep in mind this new data was already there hidden in the available data.
The key question is: does AI create new data not hidden in the available data? It does.
Examples of new data are presented by Hofstadter, The Economist, June 9th 2022.
The title of Hofstadter's paper: *Artificial neural networks are not conscious*.
Hofstadter presented 8 examples using OpenAI's GPT-3.
I present one of the examples:
Question: When was the Golden Gate Bridge transported for the second time across Egypt?
Answer GPT-3: The Golden Gate Bridge was transported for the second time across Egypt in October of 2016.
Hofstadter: *GPT-3 has no idea that it has no idea about what it is saying.*

Hence, Artificial networks today are not conscious. AI cannot create meaningful new ideas like a General Theory of Relativity. Only human beings can.
Now we walk on thin ice: the Juvenalis Dilemma.
Meaningful new ideas creating new proven data? Well, is it possible in the near future the answer to the question of the Golden Gate Bridge is considered to be meaningful new data? Then, anything goes, and we look the end of humanity right in the face.
Obviously, the Golden Gate Bridge question is a nonsense question. Is there a mechanism to prevent a nonsense answer to become new data? Is there a solution for the juvenalis dilemma?
Hypothesis: meaningful data can only be created by human beings.

16 Archimedes, The Lever, and The Method

Strogatz in Infinite Powers, describes The Method and how Archimedes dealt with Infinity(Chapter 2). This is illustrated how Archimedes did work out the area of the parabolic segment. As mentioned by Strogatz, The Method is mechanical (page 43-47). Meaning, by weighting infinite small slices of the parabolic segment. Then Strogatz told not a single copy of The Method to have survived the Middle Ages. By coincidence, the Method was found in 1998 in a medieval prayerbook. This book is a palimpsest. The quest for the palimpsest is published by Netz and Noel: *The Archimedes Codex. A scientific detective*.
Additional reading on Archimedes and The Method is the book by Napolitani(2001).
Palimpsest: a parchment, tablet, etc. that has been written upon several times, with previous, erased texts still partly visible.
Palimpsest: Adj. written upon more than once(Webster).

It's not just about a Lever, but also about a Fulcrum. Is it? A lever without a Fulcrum is not a Lever, but just another rod.

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