De Fermat’s Last Theorem, Pythagorean Triples and The Descente Infinie revisited

In Search for the Margin of Diophantus
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Abstract

Another proof of the non-existence of de Fermat’s last theorem is presented.
Methods used in the time of Fermat has been applied.
This is done by using Pythagorean Triples and Fermat’s Descent Infinie.

§1. Introduction.

‘*Fermat’s equation:* $x^n + y^n = z^n$, **(1.1)**

has no solutions for $n \geq 3'$.
A statement by Andrew Wiles written on the black board after the presentation of the conclusions concerning equation **(1.1).**
Is there another proof of the last theorem of de Fermat, fitting into the margin of de Fermat’s copy of Diophantus?
de Fermat mentioned his proof not to fit into the width of the margin of his Diophantus copy.

Ockham's razor, sort of.
This stimulated me to investigate de Fermat's last theorem.
It all started with $n = 2$ and

$$x^2 + y^2 = z^2. \qquad \textbf{(1.2)}$$

Equation **(1.2)** represents an infinite number of solutions for x, y and z as positive integers: $\{a, b \text{ and } c \in \mathbb{P}\}$, the Pythagorean Triples.
This equation, **(1.2)**, is discussed in some detail in Noordzij.
I present the results in §2 Pythagorean Triples.

§2. Pythagorean Triples

First Euclid's Formula and proof of the existence of Pythagorean triples.

Proof of Euclid's formula
For $\{m, n \in \mathbb{N}\}$:
- $a = m^2 - n^2$,
- $b = 2mn$,
- $c = m^2 + n^2$.

Then,
$a^2 + b^2 = c^2$.
Caveat: not all triples found in this way are primitive.
Set m, n to be odd $\Longrightarrow$ $a^2 + b^2 = c^2$, can be, at least, divided by 2.
To obtain primitive triples:
set m to be odd and n to be even, or the other way around $\Longrightarrow$ a is odd and c is odd, resulting into primitive triples.
End of Proof

Next, another proof for Pythagorean triples:

Proof
With **(1.2)** and $x = a, y = b$, and $z = c$,

$$c = \sqrt{a^2 + b^2}\,. \qquad \textbf{(2.1)}$$

Plug into **(2.1)**, $a = a_1 a_2$, where a_1, a_2 are products of powers of prime numbers and no common factors:

$$c = \sqrt{(a_1 a_2)^2 + b^2}\,. \qquad \textbf{(2.2)}$$

a_1 or a_2 can be chosen to be equal 1, for instance $(3,4,5)$.
If we can compose an expression for b^2 of which $(a_1 a_2)^2$ in **(2.2)** is part of a double product of b^2, then an integer c can be obtained.
So, the composition

$$b^2 = \tfrac{1}{4}(a_1^4 - 2(a_1 a_2)^2 + a_2^4) \to b = \tfrac{1}{2}|a_1^2 - a_2^2| = \tfrac{1}{2}|(a_1 - a_2)(a_1 + a_2)|\,,$$

will do the job.
Substitute $b^2 = \frac{1}{4}(a_1^4 - 2(a_1 a_2)^2 + a_2^4)$ into $\sqrt{(a_1 a_2)^2 + b^2}$, **(2.2)**,
hence, c in **(2.2)** :

$$c = \sqrt{(a_1 a_2)^2 + b^2} = \sqrt{\tfrac{1}{4}(a_1^4 + 2(a_1 a_2)^2 + a_2^4)} = \tfrac{1}{2}(a_1^2 + a_2^2). \qquad \textbf{(2.3)}$$

So,

$$a = a_1 a_2, \qquad \textbf{(2.4)}$$

$$b = \tfrac{1}{2}|(a_1^2 - a_2^2)|, \qquad \textbf{(2.5)}$$

$$c = \tfrac{1}{2}(a_1^2 + a_2^2), \qquad \textbf{(2.6)}$$

are all integers.
In addition, b is even and c is odd.
End of Proof.
Note: when we had chosen , $a = 4a_1 a_2$, Euclid's result is obtained.

§3. Notation

We assume a solution of **(1.1)** in terms of integers x, y and z : a so-called Fermat Triple. Denominate the solution a "Fermat" triple: $x = A_n, y = B_n$ and $z = C_n$.

$[A_n, B_n, C_n]$ represents a set of triples.

A_n, B_n and C_n are positive integers. A_n is odd, B_n assumed to be even and C_n must be odd. So, $\{\exists A_n, C_n \in \mathbb{P}|$ and odd$\}$ and $\{\exists B_n \in \mathbb{P}|$ and even$\}$; $\{\forall n \in \mathbb{P}|n \geq 3\}$.

A_n, B_n and C_n are relative- or co-prime.

I will use(see e.g., Spivak):

$\forall$, the al kwantor meaning : "for all". So, $\{\forall n \in \mathbb{P}\}$ →for all n belonging to the set of natural numbers larger than zero.

$\exists$, the existential kwantor: "there exists". So, $\{\exists n \in \mathbb{P}\}$ →there exists an n belonging to the set of natural numbers larger than zero.

§4. de Fermat's last Theorem.

$x^n + y^n = z^n$.

I assume the preceding equation to have a so-called de Fermat triple, $[A_n, B_n, C_n]$, as solution, for $n \geq 3$.

The existence of $[A_n, B_n, C_n]$ will be investigated in the following proof. Use is made of Pythagorean Triples, § 2 .

Proof

I rewrite **(1.1)**:

$$x^n + y^n = z^n \Leftrightarrow \left(x^{\frac{n}{2}}\right)^2 + \left(y^{\frac{n}{2}}\right)^2 = \left(z^{\frac{n}{2}}\right)^2 . \qquad \textbf{(4.1)}$$

Eq. **(4.1)** is identical to Eq.**(1.2)** .

Consequently,

$\left\{\exists \left[x^{\frac{n}{2}} = a, y^{\frac{n}{2}} = b, z^{\frac{n}{2}} = c\right] \in \mathbb{P}\right\}$ represents an infinite set of Pythagorean Triples $\{\forall n \in \mathbb{P}|n \geq 3\}$, as shown by the *Proof of Euclid*.

This set of Triples equals the triples found for $n = 2$.

If not, the proof by Euclid is not correct. A contradiction.

Hence, with the set of triples $[A_n, B_n, C_n]$:

$$A_n^n + B_n^n = C_n^n \Longrightarrow \left(A_n^{\frac{n}{2}}\right)^2 + \left(B_n^{\frac{n}{2}}\right)^2 = \left(C_n^{\frac{n}{2}}\right)^2 .$$

Consequently,

$A_n^{\frac{n}{2}} = a$,

$B_n^{\frac{n}{2}} = b$,

and

$C_n^{\frac{n}{2}} = c$,

for $\{\forall n \in \mathbb{P}|n \geq 3\}$.

For example

$$A_3^{\frac{3}{2}} = A_4^{\frac{4}{2}} = A_5^{\frac{5}{2}} = \cdots = A_n^{\frac{n}{2}} = a \Longrightarrow A_3^3 = A_4^4 = A_5^5 = \cdots = A_n^n = a^2, \qquad \textbf{(4.2)}$$

$B_3^{\frac{3}{2}} = B_3^{\frac{3}{2}} = B_5^{\frac{5}{2}} = \cdots = B_n^{\frac{n}{2}} = b \Longrightarrow B_3^3 = B_4^4 = B_5^5 = \cdots = B_n^n = b^2$, **(4.3)**
and
$C_3^{\frac{3}{2}} = C_4^{\frac{4}{2}} = C_5^{\frac{5}{2}} = \cdots = C_n^{\frac{n}{2}} = c \Longrightarrow C_3^3 = C_4^4 = C_5^5 = \cdots = C_n^n = c^2$. **(4.4)**

Obviously, solutions of above the equalities, **(4.2)- (4.4)** are:
$A_n^n = 1$, or 0,
$B_n^n = 1$, or 0,
and
$C_n^n = 1$, or 0.
Furthermore
$a = b = c = 1$,
$a = b = c = 0$.
Giving the following trivial triples:
$[a, b, c] = [1,0,1]$, $[0,1,1]$, or $[0,0,0]$,
and
$[A_n, B_n, C_n] = [1,0,1]$, $[0,1,1]$, or $[0,0,0]$.
Are there other solutions, triples?
Remark 1
Here I could have used Fermat's method of *"Descente Infinie"* to prove the non-existence of Fermat Triples for $n = 3$.

We know a^2, an integer, can be expanded in a product of powers of prime numbers, P say.
Furthermore, likewise
$A_3, A_4, A_5, \dots, A_n$,
can be expanded in a product of powers of prime numbers respectively:
$P_3, P_4, P_5, \dots, P_n$.
Then,
$A_3^3 = A_4^4 = A_5^5 = \cdots = A_n^n = a^2 \Longrightarrow P_3^3 = P_4^4 = P_5^5 = \cdots = P_n^n = P^2$. **(4.5)**
Now,
$A_3 = P^{2/3}$, $A_4 = P^{1/2}, A_5 = P^{2/5}, \dots, A_n = P^{2/n}$.
Consequently
$A_n \neq A_{n-k}$, and A_{n-k} and A_n are co-prime,
with $\{k \in \mathbb{P} | 3 \leq k < n\}$.
So,
$P_n \neq P_{n-k}$, the fundamental theorem of arithmetic,
and
$P_n^n \neq P_{n-k}^{n-k}$.
Then,
$A_n^n \neq A_{n-k}^{n-k}$.
The preceding inequality contradicts Eq.**(4.2)**.
The equality, Eq. **(4.5)**, only holds for
$P_n = P_{n-k} \Longrightarrow A_n = A_{n-k} \Longrightarrow A_n^n = A_{n-k}^n$.
With $A_n^n = A_{n-k}^{n-k}$, Eq. **(4.2),** $\Longrightarrow A_n^n = A_{n-k}^n = A_{n-k}^{n-k} \Longrightarrow$
$\Longrightarrow A_{n-k}^{n-k}\left(A_{n-k}^k - 1\right) = 0$,
we conclude no other solutions exist other than
$A_n^n = 1$, or $0 \Longrightarrow A_n = 1$, or 0.
Similarly
$B_n = 1$, or 0,
and
$C_n = 1$, or 0,
for
$\{\forall n \in \mathbb{P} | n \geq 3\}$.
Finally, we need to consider the case where
A_n is expressed in a power of a single prime number $p \geq 3$, or a product of power of prime numbers.
We use this to analyse, Eq. **(4.2)**,

$A_3^3 = A_4^4 = A_5^5 = \cdots = A_n^n = a^2$

First an example
$A_3^3 = A_4^4 = a^2$.
With
$A_3 = p^4$, and $A_4 = p^3$,
$A_3^3 = A_4^4 = a^2 \Longrightarrow p^{12} = p^{12} = a^2$.
This gives a Pythagorean Triple:
$(p^6, \frac{p^{12}-1}{2}, \frac{p^{12}+1}{2})$.
Another example, $p \neq q$, and $p > q$,
$A_3 = p^4 q^4$, and $A_4 = p^3 q^3$,
$A_3^3 = A_4^4 = a^2 \Longrightarrow (pq)^{12} = a^2$.
This results into two Pythagorean Triples:
$[(pq)^6, \frac{(pq)^{12}-1}{2}, \frac{(pq)^{12}+1}{2}]$,
and
$[(pq)^6, \frac{p^{12}-q^{12}}{2}, \frac{p^{12}+q^{12}}{2}]$.
Remark 2
Here again we could have used Fermat's method of *"Descente Infinie"* to prove the non-existence of Fermat Triples for $n = 3$.

In general
$A_3^3 = A_4^4 = A_5^5 = \cdots = A_{n-2}^{n-2} = A_{n-1}^{n-1} = A_n^n = a^2$.
With prime numbers
$A_3 = p^{2^2 \cdot 5 \cdot \ldots (n-2) \cdot (n-1) \cdot n} \Longrightarrow A_3^3 = p^{2^2 \cdot 3 \cdot 5 \cdot \ldots (n-2) \cdot (n-1) \cdot n}$,
$A_4 = p^{3 \cdot 5 \cdot \ldots (n-2) \cdot (n-1) \cdot n} \Longrightarrow A_4^4 = p^{2^2 \cdot 3 \cdot 5 \cdot \ldots (n-2) \cdot (n-1) \cdot n}$,
$\ldots.. = \cdots \ldots \ldots \ldots \ldots \ldots \ldots ..$,
$A_{n-1} = p^{2^2 \cdot 3 \cdot 4 \cdot 5 \cdot \ldots (n-2) \cdot n} \Longrightarrow A_{n-1}^{n-1} = p^{2^2 \cdot 3 \cdot 4 \cdot 5 \cdot \ldots (n-2) \cdot (n-1) \cdot n}$,
$A_n = p^{2^2 \cdot 3 \cdot 4 \cdot 5 \cdot \ldots (n-2) \cdot (n-1)} \Longrightarrow A_n^n = p^{2^2 3 \cdot 5 \cdot \ldots (n-2) \cdot (n-1) \cdot n}$,
where $2^2 \cdot 3 \cdot 5 \cdot \ldots (n-2) \cdot (n-1) \cdot n$ is the least common multiple.
Then
$A_3^3 = A_4^4 = A_5^5 = \cdots = A_{n-2}^{n-2} = A_{n-1}^{n-1} = A_n^n = a^2$.
With the Pythagorean Triple:
$(p^{2 \cdot 3 \cdot 5 \cdot \ldots (n-2) \cdot (n-1) \cdot n}, \frac{p^{2^2 \cdot 3 \cdot 5 \cdot \ldots (n-2) \cdot (n-1) \cdot n} - 1}{2}, \frac{p^{2^2 \cdot 3 \cdot 5 \cdot \ldots (n-2) \cdot (n-1) \cdot n} + 1}{2})$.
Again we obtain:
$A_n \neq A_{n-k}$, and A_{n-k} is co-prime with A_n,
with $\{k \in \mathbb{P} | 3 \leq k < n\}$.
However, $A_n^n = A_{n-k}^{n-k}$.

Remark 3 *about the least common multiple*:
In the above analysis $(n-2) \cdot (n-1) \cdot n$ is included.
Suppose $n = 50 = 2 \cdot 5^2$. Since 2 and 5^2 are already included in the least common multiple, 50 is not. 25 is included by raising the power of: $5 \rightarrow 5^2$.
$n - 1 = 49 = 7^2$. Consequently $7 \rightarrow 7^2$ and 49 is not explicitly included in the least common multiple.
$n - 2 = 48 = 2^4 \cdot 3$. Since $16 = 2^4$, 48 is not explicitly included in the least common multiple.
$n + 1 = 51 = 3 \cdot 17$. Both, 3 and 17 are already included and 51 is not explicitly included.
So, the afore mentioned considerations must be kept in mind by interpreting:
$2^2 \cdot 3 \cdot 5 \cdot \ldots (n-2) \cdot (n-1) \cdot n$.
The above analysis can be used for the product of prime numbers as presented above: instead of p, pq can be used.

Let us investigate
$A_3 = p^2 \Longrightarrow A_3^3 = p^6 = a^2$.
The Pythagorean Triple is:

$(p^3, \frac{p^6-1}{2}, \frac{p^6+1}{2})$.
Hence, with Eq.**(4.4)**,
$C_3^{3/2} = c = \frac{p^6+1}{2} \Longrightarrow C_3 = (\frac{p^6+1}{2})^{2/3}$.
Can C_3 be an integer?
Now we need Fermat's method of *"Descente Infinie"* to use the non-existence of Fermat Triples for $n = 3$: no Fermat triple $[A_3, B_3, C_3]$, except trivial triples.
So, investigating
$A_3^3 + B_3^3 = C_3^3$,
assuming A_3, to be an integer, B_3 and/ or C_3 are no integers→the triple $[A_3, B_3, C_3]$ does not exists.
Eqs. **(4.3)-(4.4)** are contradicted?
Let us examine Eq. **(4.4)** :
$C_3^{\frac{3}{2}} = C_4^{\frac{4}{2}} = C_5^{\frac{5}{2}} = \cdots = C_n^{\frac{n}{2}} = c \Longrightarrow C_3^3 = C_4^4 = C_5^5 = \cdots = C_n^n = c^2$.
We construct the following table on the basis of $A_3^3 = p^m$,
where m represents the least common multiple $2^2 \cdot 3 \cdot 5 \cdot \ldots (n-2) \cdot (n-1) \cdot n$ and
$C_n^{n/2} = \frac{p^m+1}{2}$:
The table

$n = 3$	4	5	...	n		
C_3^3	C_4^4	C_5^5	...	C_n^n	$(\frac{p^m+1}{2})^2$	$= integer$
$C_3^{3/2}$	$C_4^{4/2}$	$C_5^{5/2}$	...	$C_n^{n/2}$	$\frac{p^m+1}{2}$	$= integer$
$C_3^{3/3}$	$C_4^{4/3}$	$C_5^{5/3}$	...	$C_n^{n/3}$	$(\frac{p^m+1}{2})^{2/3}$	$\neq integer$
$C_3^{3/4}$	$C_4^{4/4}$	$C_5^{5/4}$	...	$C_n^{n/4}$	$(\frac{p^m+1}{2})^{1/2}$	$\neq integer$
$C_3^{3/5}$	$C_4^{4/5}$	$C_5^{5/5}$		$C_n^{n/5}$	$(\frac{p^m+1}{2})^{2/5}$	$\neq integer$

In the third row of the above table
$C_3^{3/3} \equiv C_3$ appears and due to the non-existence of the Fermat Triple $[A_3, B_3, C_3]$, $C_3 \neq integer$.
What about
$C_3^{3/4}$ an integer? If so, the possibility exists $C_3[= (integer)^{\frac{4}{3}}]$ equals an integer. This contradicts $C_3 \neq integer$. Consequently, $C_3^{3/4} \neq integer \rightarrow C_4^{\frac{4}{4}} \equiv C_4 \neq integer$, the fourth row in the table, etc.
Conclusion,
For $x^n + y^n = z^n \Leftrightarrow (x^{n/2})^2 + (y^{n/2})^2 = (z^{n/2})^2$, and
and $\{\forall n \in \mathbb{P} | n \geq 3\}$, no primitive set of Triples can be found ∴ the assumption: a so-called de Fermat triple, $[A_n, B_n, C_n]$, exists, is contradicted. Only a set trivial triples $[A_n, B_n, C_n] = [1,0,1], [0,1,1]$, or $[0,0,0]$, are found

End of Proof.

§5 Do we need the Descente Infinie?

§5.1 $a^2=p^m$ and Pythagorean Triples.

Let us investigate $n = 3$ and imply the results of Pythagorean Triples.
Keep in mind, $A_3^3 = a^2 = p^m$,
where m is the least common multiple.

Then,
$C_3^{3/2} = \frac{p^m+1}{2}$, an integer and odd,
$B_3^{3/2} = \frac{p^m-1}{2}$, an integer and even.
Hence,
$C_3^{\frac{3}{2}} - B_3^{\frac{3}{2}} = 1.$
We assume
C_3 to be an integer, $I, I \geq 3$
and
B_3 to be an integer, $I_1 \leq I - 1 \geq 2.$
For the analysis we use
$I = 3,$
and
$I_1 = 2.$
Then,
$C_3^{\frac{3}{2}} - B_3^{\frac{3}{2}} = (I^3)^{\frac{1}{2}} - [(I-1)^3]^{\frac{1}{2}} = 27^{\frac{1}{2}} - 8^{\frac{1}{2}} > 1.$
We assumed the de Fermat's Triple to exits.
Consequently
$C_3^{\frac{3}{2}} - B_3^{\frac{3}{2}} = 1,$
is contradicted → B_3 and C_3 are no integers→ de Fermat's Triple does not exist for $n = 3.$
Notice: for $\{r \in \mathbb{P} | r \geq 3\} \rightarrow r^3 - (r-1)^3 > 1$.

- $n = 4$.
$C_4^{4/2} = \frac{p^m+1}{2}$, an integer and odd,
$B_4^{4/2} = \frac{p^m-1}{2}$, an integer and even.
$C_4^{\frac{4}{2}} - B_4^{\frac{4}{2}} = 1.$
We assume
C_4 to be an integer, $I, I \geq 3$
and
B_4 to be an integer, $I_1 \leq I - 1 \geq 2.$
For the analysis we use
$I = 3,$
and
$I_1 = 2.$
Then,
$C_4^{\frac{4}{2}} - B_4^{\frac{4}{2}} = I^2 - (I-1)^2 = 9 - 4 > 1.$
We assumed the de Fermat's Triple to exits.
Consequently

$C_4^{\frac{4}{2}} - B_4^{\frac{4}{2}} = 1,$
is contradicted → B_4 and C_4 are no integers→ de Fermat's Triple does not exist for $n = 4$.

$n = 5$.
$C_5^{5/2} = \frac{p^m+1}{2}$, an integer and odd,
$B_5^{5/2} = \frac{p^m-1}{2}$, an integer and even.
$C_5^{\frac{5}{2}} - B_5^{\frac{5}{2}} = 1.$
We assume
C_5 to be an integer, $I, I \geq 3$
and
B_5 to be an integer, $I_1 \leq I - 1 \geq 2$.
For the analysis we use
$I = 3,$
and
$I_1 = 2.$
Then,
$C_5^{\frac{5}{2}} - B_5^{\frac{5}{2}} = (I^5)^{\frac{1}{2}} - [(I-1)^5)]^{\frac{1}{2}} = 9\sqrt{3} - 4\sqrt{2} > 1.$
We assumed the de Fermat's Triple to exits.
Consequently
$C_5^{\frac{5}{2}} - B_5^{\frac{5}{2}} = 1,$
is contradicted → B_5 and C_5 are no integers→ de Fermat's Triple does not exist for $n = 5$.

- n
Suppose
$C_n^{\frac{n}{2}} - B_n^{\frac{n}{2}} = (I^n)^{\frac{1}{2}} - [(I-1)^n)]^{\frac{1}{2}} > 1.$
We assumed the de Fermat's Triple to exits.
Consequently
$C_n^{\frac{n}{2}} - B_n^{\frac{n}{2}} = 1,$
is contradicted → B_n and C_n are no integers→ de Fermat's Triple does not exist for n.

- $n + 1$?
$C_{n+1}^{\frac{n+1}{2}} - B_{n+1}^{\frac{n+1}{2}} = (I^{n+1})^{\frac{1}{2}} - [(I-1)^{n+1})]^{\frac{1}{2}} = I^{\frac{1}{2}} (I^n)^{\frac{1}{2}} - (I-1)^{\frac{1}{2}}[(I-1)^n]^{1/2} >$
$> I^{\frac{1}{2}}\{(I^n)^{\frac{1}{2}} - [(I-1)^n)]^{\frac{1}{2}}\} > 1.$

Hence , we conclude the Pythagorean Triples to be sufficient to prove the de Fermat's Triple not to exist for $n, \{n \in \mathbb{P} | n \geq 3\}$ with $a = p^m$.
a constitute the Pythagorean Triple $[a, b, c]$, p is a prime number ≥ 3 and m is the least common multiplier, §4.

§5.2 $a^2=p^m q^m$, Pythagorean Triples and The Descente Infinie

In §4, $A_3^3 = \cdots . = A_n^n = a^2 = p^m q^m$, has been mentioned.
Again, m is the least common multiple.
There, two Pythagorean Triples are presented:

$[(pq)^6, \frac{(pq)^{12}-1}{2}, \frac{(pq)^{12}+1}{2}]$,

and

$[(pq)^6, \frac{p^{12}-q^{12}}{2}, \frac{p^{12}+q^{12}}{2}]$.

Using the first triple, the analysis of the above §5.1 can be used.
For the second triple the Method of Descente Infinie is needed as applied in §4.
Furthermore, depending on p and q, the second triple represents more than 1 triple.

§6 $n > 3$ and the Method of Descente Infinie applied.

In paragraph §4 the method of Descente Infinie has been used for the first term, $n = 3$, in the Eqs. **(4.3)-(4.4)**.
In §5 we proved the Pythagorean Triples to be sufficient.
Can we use the method of de Fermat for $n > 3$, (Noordzij)?
We want to investigate the possible existence of the Fermat Triples $[A_n, B_n, C_n]$, for $n > 3$.
We have the equation

$$A_n^n + B_n^n = C_n^n \ . \quad \textbf{(5.1)}$$

Similarly to the procedure in § 4, we rewrite Eq. **(5.1)**

$$(A_n^{n/3})^3 + (B_n^{n/3})^3 = (C_n^{n/3})^3. \quad \textbf{(5.2)}$$

With the result of the '*Descente infinie*' (Giorello, G., et al) for a given integer $A_n^{n/3}$, $n > 3 \rightarrow$
$\rightarrow \{\nexists(B_n^{n/3}, C_n^{n/3}) \in \mathbb{P} | n > 3\}$.
Consequently,
$\{\nexists(B_n, C_n) \in \mathbb{P} | n > 3\}$.

Proof
$\{\nexists(B_n, C_n) \in \mathbb{P} | n > 3\}$.
Suppose
$\{\exists(B_n, C_n) \in \mathbb{P} | n > 3\}$.
Then,
$\{\exists(B_n^n, C_n^n) \in \mathbb{P} | n > 3\}$.
So, the possibility exists
$\{\exists(B_n^{n/3}, C_n^{n/3}) \in \mathbb{P} | n > 3\}$, Eq.**(5.2)**.
This possibility contradicts the method of the '*Descente infinie*'.
Hence,
$\{\nexists(B_n, C_n) \in \mathbb{P} | n > 3\}$.
End of Proof.

§7. Conclusions.

The nonexistence of a Fermat Triple, $\boldsymbol{n \geq 3}$, has been proven in §4 applying the existence of an infinite number of Pythagorean Triples and Fermat's Descente Infinie.
In the above analysis, we used Fermat's Descente Infinie in Eqs. **(4,2)-(4.4)**, for the first terms A_3^3 , B_3^3 and C_3^3 in Eqs. **(4,2)-(4.4)**.
In §5 it has been proven the Pythagorean Triples to be sufficient.
In § 6, the method of Descente Infinie can be used for $n > 3$.

For further reading on de Fermat's last theorem I like to mention Simon Singh's book.
Finally, to conclude the above approach on de Fermat's last theorem I also cite Feynman on de Fermat's last theorem: "*For my money Fermat's theorem is true*". Feynman estimated that the probability of finding integer solutions is less than 10^{-200} (Schweber).

§8. Literature.

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