From De Fermat to Pythagoras and Back.

The emergence of Format's Last Theorem from Pythagorean Triples
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# Content



# Abstract

Another proof of De Fermat's last theorem is presented.
A concise proof of De Fermat's last theorem, is obtained by using Pythagorean Triples.
Use has been made of the proof by contradiction.

Tags: Culture, Descente Infinie, Diophantus, Education, De Fermat, Fundamental, Ground State Triples, Theorem of Arithmetic, History, Mathematics, Prime Numbers, Proof by Contradiction, Pythagorean Triples, Andrew Wiles.

# §1. Introduction.

'*Fermat's equation:* $a^n+b^n = c^n$ , **(1.1)**

*has no solutions* $\{a, b, c \text{ and } n \in \mathbb{N} | n \geq 3\}$
A statement by Andrew Wiles written on the black board after the presentation of the conclusions concerning equation **(1.1)**, Giorello, G. and C. Sinigaglia

Is there another proof of the last theorem of de Fermat, fitting into the margin of de Fermat's copy of Diophantus?
De Fermat mentioned his proof not to fit in the width of the margin of his Diophantus copy. An Ockham's razor, sort of. This stimulated me to investigate de Fermat's last theorem. Which tools could have been used by De Fermat?
Well, he could have used the analysis of Pythagoras and Euclid. In addition we used the proof by contradiction: reductio ad absurdum. The form of argument that attempts to establish a claim by showing that the opposite scenario would lead to absurdity or contradiction,
https://en.wikipedia.org

# §2 From De Fermat to Pythagoras.

## §2.1 The elements of the Pythagorean Triples

In this paragraph $n = 2$ in **(1.1)** will be analysed. For a given value of $a$, solutions for $b$ and $c$are obtained, the so-called Pythagorean Triples. See also Noordzij(1).
I assume the existence of a solution of **(1.1)** denominated a "Pythagorean Triple":
$[\,a, b, c]$, $\{b \in \mathbb{N} | b > 0$ and even$\}$ and $\{a, c \in \mathbb{N} | a, c > 0$ and odd$\}$.
Next, we factorize $a = a_1 a_2$, where $a_1, a_2$ are products of powers of prime numbers and no common factors. Here, I assume $a$ to be odd as mentioned above and $a_1 > a_2$.
Obviously, with
$a = a_1 \rightarrow a_2 = 1$, a "ground state" of Pythagorean Triples for a particular $a$ number is obtained.
With **(1.1)**:

$$c = \sqrt{(a_1 a_2)^2 + b^2}\,. \qquad \textbf{(2.1)}$$

If I can compose an expression for $b^2$ of which $(a_1 a_2)^2$ in **(2.1)** is part of a cross product of $b^2$, then an integer $c$ can be obtained. Hence, a relation between $a$ and $b$ has to be constructed. There are no triples for randomly chosen values of the integers $a$ and $b$ Noordzij(2).
The composition, relation between $a$ and $b$:

$$b^2 = \frac{1}{4}(a_1^4 - 2(a_1 a_2)^2 + a_2^4) \rightarrow b = \frac{1}{2}(a_1^2 - a_2^2)\,,$$

will do the job.
Substitute $b^2 = \frac{1}{4}[a_1^4 - 2(a_1 a_2)^2 + a_2^4]$ into $(a_1 a_2)^2 + b^2 \rightarrow$

$$\rightarrow (a_1 a_2)^2 + \frac{1}{4}[a_1^4 - 2(a_1 a_2)^2 + a_2^4] = \frac{1}{4}[a_1^4 + 2(a_1 a_2)^2 + a_2^4].$$

Hence, $c$ in **(2.1)** :

$$c = \sqrt{(a_1 a_2)^2 + b^2} = \sqrt{\frac{1}{4}(a_1^4 + 2(a_1 a_2)^2 + a_2^4)} = \frac{1}{2}(a_1^2 + a_2^2). \qquad \textbf{(2.2)}$$

So,
with $a = a_1 a_2$ **(2.3)**

$$b = \frac{1}{2}(a_1^2 - a_2^2) = \frac{1}{2}(a_1 + a_2)(a_1 - a_2), \qquad \textbf{(2.4)}$$

$c = \frac{1}{2}(a_1^2 + a_2^2) = \frac{1}{2}(a_1 + a_2)^2 - a,$ **(2.5)**

are both integers.

In addition, as shown by **(2.4)** and **(2.5)**, $b$ is even and $c$ is odd.

In Noordzij(2, §2), basically the same result is presented as in **(2.4)** and **(2.5)**.

There Pythagoras equation is presented as

$(c - b)(c + b) = a^2 \rightarrow (c - b)(c + b) = a_2^2 \cdot a_1^2.$

In order to find out about Pythagorean triples we assumed

$(c - b) = a_2^2$ ,

and

$(c + b) = a_1^2.$

This result in the equations **(2.4)** and **(2.5).**

An example $a = 15$:

- $a_1 = 15$ , and $a_2 = 1.$

Then, with **(2.4)** and **(2.5)**

$b = 112$, and $c = 113.$

- $a_1 = 5$ , and $a_2 = 3.$

Then, with **(2.4)** and **(2.5)**

$b = 8$, and $c = 17.$

So, choose $a = 15$ and you will find two sets of triples: $[15,112,113]$ and $[15,8,17]$.

Remark:

Look at **(2.4)-(2.5)**:

These expressions look familiar $\Longrightarrow$ similar to Euclid's formula to obtain Pythagorean triples.

## §2.1 Euclid's Formula for Pythagorean Triples

See: *Formulas for generating Pythagorean Triples*, ( www.en.wikipedea.org )

For $\{m, n \in \mathbb{N}\}$:

- $a = m^2 - n^2,$
- $b = 2mn,$
- $c = m^2 + n^2.$

Then,

$a^2 + b^2 = c^2.$

Caveat: not all triples found in this way are primitive.

Set $m, n$ to be odd $\Longrightarrow$ $a^2 + b^2 = c^2$, can be, at least, divided by 2.

To obtain primitive triples:

set $m$ to be odd and $n$ to be even, or the other way around $\Longrightarrow$ $a$ is odd and $c$ is odd, resulting into primitive triples.

Note: when we had chosen in the preceding paragraph, $a = 4a_1a_2$, Euclid's result is obtained.

In addition we answer the following question: can another Triple-sequence like 3,4 and 5 be found? Let us look into it:

With

$a^2 + (a + 1)^2 = (a + 2)^2 \rightarrow (a - 3)(a + 1) = 0 \rightarrow a = 3$ .

Consequently there is just one sequence:
$a = 3, b = 4$, and $c = 5$.

## §2.2 Pythagorean Triples and De Fermat's Last Theorem

In addition, use can be made of De Fermat's Last Theorem to obtain more information on Pythagorean Triples. De Fermat's last theorem being correct, leads to the conclusion no Pythagorean triples to be found constituted of powers of integers.
An example:
Assume the triple to be $[a = x^2, b = y^2, c = z^2]$, $\{x, y, z \in \mathbb{N}| > 1\}$.
Then the Pythagorean equation and De Fermat's last theorem gives:
$(x^2)^2 + (y^2)^2 = (z^2)^2 \rightarrow x^4 + y^4 \neq z^4$,
QED.

Next, the other way around. Without the knowledge of the existence of De Fermat's Theorem:
does a Pythagorean Triple $[a = x^s, b = y^s, c = z^s]$, with $\{x, y, z$ and $s \in \mathbb{N}| \ > 1\}$ and $x$ prime, exist?

*Proof:*
We use the results from the proof of the existence of Pythagorean triples in §1.2, eqs. **(2.4)** and **(2.5)** with $a_2 = 1$.[1]
Then
- $z^s - y^s = 1$,
and
- $z^s + y^s = x^{2s}$.

$z^s - y^s = 1$, cannot be true. Obviously, for $y = 0$ a trivial Triple is found: $[1.0,1]$.
Take a closer look. Since, $\{x, y, z$ and $s \in \mathbb{N}|$ and $> 1\}$ and with $\{i \in \mathbb{N} \cup \emptyset\}$

$$z = y + 2i + 1 \rightarrow z^s = (y + 2i + 1)^s = \sum_{k=0}^{s} \binom{s}{k} y^{s-k}(2i + 1)^k \rightarrow$$
$$z^s - y^s = \sum_{k=1}^{s} \binom{s}{k} y^{s-k}(2i + 1)^k,$$

with
$y > 1$, in $\sum_{k=1}^{s} \binom{s}{k} y^{s-k}(2i + 1)^k \rightarrow z^s - y^s > 1$, indeed.
$z^s - y^s = 1$, is contradicted.
Then, the assumption Pythagorean triple $[a = x^s, b = y^s, c = z^s]$, to exist is contradicted.
Consequently, a Pythagorean triple $[a = x^s, b = y^s, c = z^s]$ cannot be found.
So,
$$(x^s)^2 + (y^s)^2 \neq (z^s)^2 \rightarrow x^{2s} + y^{2s} \neq z^{2s}, \qquad \textbf{(2.6)}$$
this expression looks familiar→ De Fermat's last theorem.
*End of Proof.*

[1] An example: $x^s = 27 \rightarrow a_2 = 1$ creates a new Triple. Factorization $a_1 = 9$, and $a_2 = 3$, results in no new Triple.

I will pay attention to the choice of $a_2 = 1^2$ , as illustrated in footnote 1.
It is about the factorization of $a = x^s$:
$x^s \cdot 1;\ x^{s-1} \cdot x;\ x^{s-2} \cdot x^2;\ \ldots\ldots;\ x^{s-i} \cdot x^i;\ \ldots\ldots;\ x^{s-k} \cdot x^k$.
For $s$ is even, the factorization stops at $s - k = k$. Obviously, a trivial triple is obtained:
$b = \frac{1}{2}\left(x^{2(s-k)} - x^{2k}\right) = 0,$
$c = \frac{1}{2}\left(x^{2(s-k)} + x^{2k}\right) = x^{2k}.$
So, the lowest possible ground state is $i = k - 2$ .
For $s$ is odd, the factorization stops at $i = k - 1$.
What about the choice $a_2 = 1$?
I shall illustrate this choice for the factorization $x^{s-i} \cdot x^i$, using eqs. **(2.4)** and **(2.5)**:
the Triple is: $\left[x^s, \frac{1}{2}\left(x^{2(s-i)} - x^{2i}\right), \frac{1}{2}\left(x^{2(s-i)} + x^{2i}\right)\right]$.
For $s$ is even:
I choose the largest possible value of $i = \frac{s}{2} - 1$:
the triple is $\left[x^s, \frac{1}{2}(x^{s+2} - x^{s-2}), \frac{1}{2}(x^{s+2} + x^{s-2})\right] = x^{s-2}[x^2, (x^4 - 1), (x^4 + 1)]$. .
Hence, with $a = x^s$, it is sufficient to analyse the factorization $x^s \cdot 1$. The remaining couples of factors, do not create new Triples with $x^s \geq x^2$.
For $s$ is odd:
Again, I choose the largest possible value of $i = \frac{1}{2}(s - 1)$:
the triple is $\left[x^s, \frac{1}{2}(x^{s+1} - x^{s-1}), \frac{1}{2}(x^{s+1} + x^{s-1})\right] = x^{s-1}[x, (x^2 - 1), (x^2 + 1)]$
Note, an example :
$x^s = 243 = 3^5$ :
the ground state
- $a = a_1 = 243$, and $a_2 = 1$.
Factorization:
- $a_1 = 81, a_2 = 3$ →the triple is
$\left[3^5, \frac{1}{2}\left(3^{2(5-1)} - 3^2\right), \frac{1}{2}\left(3^{2(5-1)} + 3^2\right)\right] = 3^2[3^3, \frac{1}{2}(3^6 - 1), \frac{1}{2}(3^6 + 1)] \rightarrow$
the ground state of 27 → $[3^3, \frac{1}{2}(3^6 - 1), \frac{1}{2}(3^6 + 1)]$;
- $a_1 = 27, a_2 = 9$ →the triple is
$\left[3^5, \frac{1}{2}\left(3^{2(5-2)} - 3^4\right), \frac{1}{2}\left(3^{2(5-2)} + 3^4\right)\right] = 3^4[3, \frac{1}{2}(3^2 - 1), \frac{1}{2}(3^2 + 1)] \rightarrow$
the ground state of 3 → $\left[3, \frac{1}{2}(3^2 - 1), \frac{1}{2}(3^2 + 1)\right] = [3,4,5]$.

Next, we investigate another Triple: $[a = x^{\frac{s}{2}}, b = y^{\frac{s}{2}}, c = z^{\frac{s}{2}}]$,
with $\{x, y, z$ and $s \in \mathbb{N} | x, y, z > 1, s > 2\}$.

*Proof*

Again, we use the results from the proof of the existence of Pythagorean triples in §2.1,

[2] This choice is an important one: $a = a_1 \cdot 1$ . I denoted this to be the ground state, an expression borrowed from quantum mechanics. A better expression is: *base case*? Without $a = a_1 \cdot 1$ , there is **no Triple**.

eqs. **(2.4)** and **(2.5)** with $a_2 = 1$.[3]
Then
- $z^{s/2} - y^{s/2} = 1$.
$z^{s/2} - y^{s/2} = 1$, cannot be true, since $z \geq y + 1$.
Consequently,
$z^{s/2} - y^{s/2} > 1$,
and the Triple $[x^{\frac{s}{2}}, y^{\frac{s}{2}}, z^{\frac{s}{2}}]$ does not exist[4].
Hence,

$$(x^{s/2})^2 + (y^{s/2})^2 \neq \left(z^{s/2}\right)^2 \rightarrow x^s + y^s \neq z^s, \qquad \textbf{(2.7)}$$

this expression looks familiar again.
Obviously, for $y = 0$ a trivial Triple is found: $[1,0,1]$.
*End of Proof*

## §2.3 The number of triples for a given $a$.

Conjecture: for a given $a$ the number of triples equals $2^{p-1}$,
where $a$ is factorised into $p$, different, prime numbers. Set $n = 2$ in eq. **(1.1).**
Examples, using **(2.3)**-(**2.5)**:
- $a = 3 \rightarrow [3,4,5]$,
- $a = 5 \rightarrow [5,12,13]$,
- $a = 7 \rightarrow [7,24,25]$,
- $a = 9 \rightarrow [9,40,41]$,
- $a = 11 \rightarrow [11,60,61]$,
- $a = 13 \rightarrow [13,84,85]$,
- $a = 15 \rightarrow [15,112,113]$ and $[15,8,17]$. Prime numbers 3 and 5. $p = 2$.

Next we choose $a$ to be factorized into 3,5 and 7, $p = 3$. This produces the following four triples
- $a = 105 \rightarrow [105,5512,5513], [105,608,617], [105,208,233]$, and $[105,68,137]$.

Then $a$ to be factorised into 3,5,7 and 11, $p = 4$ giving the following eight values for $a_1$ and $a_2$

| $a_1$ | $a_2$ |
|---|---|
| $3 \cdot 5 \cdot 7 \cdot 11$ | $1$ |
| $5 \cdot 7 \cdot 11$ | $3$ |
| $3 \cdot 7 \cdot 11$ | $5$ |
| $3 \cdot 5 \cdot 11$ | $7$ |
| $3 \cdot 5 \cdot 7$ | $11$ |
| $7 \cdot 11$ | $3 \cdot 5$ |
| $5 \cdot 11$ | $3 \cdot 7$ |
| $5 \cdot 7$ | $3 \cdot 11$ |
| | |

[3] $x^s = 27 \rightarrow a_2 = 1$ creates a new Triple. Factorization $a_1 = 9$, and $a_2 = 3$, results in no new information.
[4] Caveat tres: keep in mind for $x$ being a quadratic natural number, $x^{\frac{s}{2}}$ represents a natural number.

Finally, let us factorize $a$ into 3,5,7,11 and 13, $p = 5$ giving the following 16 values for $a_1$ and $a_2$

| $a_1$ | $a_2$ | | $a_1$ | $a_2$ |
|---|---|---|---|---|
| $3 \cdot 5 \cdot 7 \cdot 11 \cdot 13$ | $1$ | | $3 \cdot 11 \cdot 13$ | $5 \cdot 7$ |
| $5 \cdot 7 \cdot 11 \cdot 13$ | $3$ | | $3 \cdot 7 \cdot 13$ | $5 \cdot 11$ |
| $3 \cdot 7 \cdot 11 \cdot 13$ | $5$ | | $3 \cdot 7 \cdot 11$ | $5 \cdot 13$ |
| $3 \cdot 5 \cdot 11 \cdot 13$ | $7$ | | $3 \cdot 5 \cdot 13$ | $7 \cdot 11$ |
| $3 \cdot 5 \cdot 7 \cdot 13$ | $11$ | | $3 \cdot 5 \cdot 11$ | $7 \cdot 13$ |
| $3 \cdot 5 \cdot 7 \cdot 11$ | $13$ | | $11 \cdot 13$ | $3 \cdot 5 \cdot 7$ |
| $7 \cdot 11 \cdot 13$ | $3 \cdot 5$ | | | |
| $5 \cdot 11 \cdot 13$ | $3 \cdot 11$ | | | |
| $5 \cdot 7 \cdot 13$ | $3 \cdot 13$ | | | |

Set $13 \to 1$ and $p = 4$ is obtained.
Now I consider the conjecture to be a theorem making use of the proof by "reduction".

## §3 .....and Back to De Fermat's Last Theorem

See also De Fermat's Last Theorem, Pythagorean Triples and The Descente Infinie revisited, www.leennoordzij.me , Noordzij(1).

There are no triples for randomly chosen values of the integers $a$ and $b$ Noordzij(3). A relation between $a, b$ and $c$ in **(1.1)** is necessary. For this relation, use will be made of the existence of Pythagorean Triples.
Furthermore, factorization of

$$a^n + b^n = c^n \to$$

$$\to (c-b)\left(\sum_{k=0}^{n-1} c^{n-1-k}\, b^k\right) = a_2^n \cdot a_1^n, \qquad \textbf{(3.1)}$$

with the presumption $a = a_1 \cdot a_2$, and $\{a, b, c \in \mathbb{N} | > 0, a$ and $c$ odd, $b$ even$\}$, $\{n \in \mathbb{N} | n > 2\}$.

*Proof*
$(c-b)\left(\sum_{k=0}^{n-1} c^{n-1-k}\, b^k\right) = c^n - b^n$ :
$(c-b)\left(\sum_{k=0}^{n-1} c^{n-1-k}\, b^k\right) = \sum_{k=0}^{n-1} c^{n-k}\, b^k - \sum_{k=0}^{n-1} c^{n-1-k}\, b^{k+1} \to$
$\to \; c^n + \sum_{k=1}^{n-1} c^{n-k}\, b^k - b^n - \sum_{k=0}^{n-2} c^{n-1-k}\, b^{k+1}$ .
Now change the summation index of $\sum_{k=1}^{n-1} c^{n-k}\, b^k$ , into $k - 1 = r$ and we have:
$c^n - b^n + \sum_{r=0}^{n-2} c^{n-1-r}\, b^{r+1} - \sum_{k=0}^{n-2} c^{n-1-k}\, b^{k+1}$ .
Hence
$\sum_{r=0}^{n-2} c^{n-1-r}\, b^{r+1} - \sum_{k=0}^{n-2} c^{n-1-k}\, b^{k+1} = 0$ .
Consequently $(c-b)\left(\sum_{k=0}^{n-1} c^{n-1-k}\, b^k\right) = c^n - b^n$ .
*End of Proof.*

With the presumption of $a, b$ and $c$ being a “Fermat” triple,
$(c-b)$ and $(\sum_{k=0}^{n-1} c^{n-1-k}\, b^k)$
are positive integers. Both $(c-b)$ and $(\sum_{k=0}^{n-1} c^{n-1-k}\, b^k)$ are integer factors of $a^n$, consisting of products of powers of prime numbers.

*Proof*:
Let $(c-b)$ be no integer factor of $a^n$.
Consequently $\frac{a^n}{(c-b)}$ is not an integer.
Then
$\sum_{k=0}^{n-1} c^{n-1-k}\, b^k)$ cannot be an integer.
For a Fermat triple $(c-b)$ has to be a factor of $a^n$ and has to be an integer.
Hence,
$\sum_{k=0}^{n-1} c^{n-1-k}\, b^k$, is an integer factor of $a^n$.
*End of proof.*

So, $(c-b)$ to be an integer is a condition to find Fermat triples.
In the following we will use $c-b=a_2^n$ .

De Fermat’s theorem in a nutshell
$a^n+b^n \neq c^n$,
with
$\{a, b, c \in \mathbb{N}|$ and $>0\}$ and $\{n \in \mathbb{N}|n>2\}$.
In §2.1 we obtained two examples of De Fermat’s Last Theorem: eqs. **(2.6)** and **(2.7)**.

Let us look once more into De Fermat’s equation and transform De Fermat’s equation into the Pythagorean equation:
$a^n+b^n=c^n \rightarrow (a^{n/2})^2+(b^{n/2})^2=(c^{n/2})^2$, (see **(2.7)** ).
In the following the proof of De Fermat’s Last Theorem is presented.
Use is made of proof by contradiction of $\{a, b, c \in \mathbb{N}|$ and $>1\}$.

*Proof*
$(a^{n/2})^2+(b^{n/2})^2=(c^{n/2})^2$, **(3.2)**
represents the Pythagorean equation, where $[a^{\frac{n}{2}}, b^{\frac{n}{2}}, c^{\frac{n}{2}}]$ is presumed to constitute a Pythagorean Triple for any integer $n \geq 3$ [5].

| |
|---|
| Caveat Tres: I presume $\left[a^{\frac{n}{2}}, b^{\frac{n}{2}}, c^{\frac{n}{2}}\right]$ to be Pythagorean Triples. In the following, to test this presumption, the method of reductio ad absurdum( the method by contradiction) is used. Obviously it is easily shown, e.g., $a^{\frac{n}{2}}$ is not a natural number for a particular value of $n$. This is necessary, however not sufficient. Since, with $n=4$, $a^2$ is a natural number. |

[5] For $n=3$, De Fermat showed the non-existence of so-called De Fermat’s Triples using the method of *Descente Infinie,* Noordzij(1).

The questions to be answered:
- can $b$ and/0r $c$ be integers, for $n > 2$ ?
- and do the triples exist?

In §2.1 we analysed the Pythagorean Triple $[x^{\frac{s}{2}}, y^{\frac{s}{2}}, z^{\frac{s}{2}}]$ with $x$ prime. This Triple does not exist.
We presume $\{a, b, c \in \mathbb{N}|$ and $> 1\}$ and $\{n \in \mathbb{N}|n > 2\}$,
and the Pythagorean Triple in **(3.2)**
$[a^{\frac{n}{2}}, b^{\frac{n}{2}}, c^{\frac{n}{2}}]$ to exist.
A claim to be tested.
We use the results from the proof of the existence of Pythagorean triples in §2.1,
eqs. **(2.4)** and **(2.5)** with $a^{n/2} = a_1^{n/2} \cdot a_2^{n/2}$:
$c^{\frac{n}{2}} - b^{\frac{n}{2}} = a_2^n$.[6]
Using **(3.1)**
$c - b = a_2^n \rightarrow c^{\frac{n}{2}} = (b + a_2^n)^{\frac{n}{2}}$.
Now, we have obtained two contradicting expressions:
$c^{\frac{n}{2}} = b^{\frac{n}{2}} + a_2^n$,
and
$c^{\frac{n}{2}} = (b + a_2^n)^{\frac{n}{2}}$.
In the table below we present the consequences:

| | |
|---|---|
| Suppose<br>$c^{\frac{n}{2}} = b^{\frac{n}{2}} + a_2^n$,<br>to be true, then | Suppose<br>$c^{\frac{n}{2}} = (b + a_2^n)^{\frac{n}{2}}$ ,<br>to be true, then |
| $c^{\frac{n}{2}} = (b + a_2^n)^{\frac{n}{2}}$,<br>cannot be true. | $c^{\frac{n}{2}} = b^{\frac{n}{2}} + a_2^n$,<br>cannot be true. |
| Consequently $b$ and/or $c$ in<br>$a^n + b^n = c^n$,<br>cannot be integers(natural numbers). | Consequently<br>$(a^{n/2})^2 + (b^{n/2})^2 \neq (c^{n/2})^2$.<br>The triples do not exist. |
| $\therefore\ a^n + b^n \neq c^n$. | $\therefore\ a^n + b^n \neq c^n$ |

Then, for a given integer(natural number) $a$:
$b$ and/or $c$ are irrational.
In Noordzij(4), the above table is discussed in more detail and is denominated
**The Table of contradictions**

To summarize
De Fermat's Last Theorem shows itself using Pythagorean Triples and De Fermat's Last Theorem is correct:
the claim $\{a, b, c \in \mathbb{N}|$ and $> 0\}$ and $a^n + b^n = c^n$ for $n > 2$ is tested and rejected using

[6] We could have set $a_2^n = 1$. This case is denoted the Ground State Triple. There is no need to do that.

reductio ad absurdum.
*End of Proof*

## §4 Conclusions

De Fermat's Theorem is found to be correct using the approach via the route of Pythagorean Triples. Or: "Fermat's Last Theorem hidden in Pythagorean Triples".
The expressions used for the elements of the Pythagorean Triples are derived in §2.1, expressions **2.4** and **2.5**.
In §3 we transformed De Fermat's equation into the Pythagorean equations and finally obtained

$$(a^{n/2})^2 + (b^{n/2})^2 \neq (c^{n/2})^2 \to a^n + b^n \neq c^n,$$

proving De Fermat's Last Theorem to be correct.
Whether or not this paper represents the tools used by De Fermat remains unknown, alas.

## §5 Literature


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